

# **PROPORTIONAL SAMPLING**

**Presented  
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# NOTATION

$P \rightarrow$  Population proportion of members belonging to the class  $C$ .

$p \rightarrow$  Sample proportion of members belonging to the class  $C$ .

$Y_\alpha = 1, \text{if } U_\alpha \in C; \text{ and } Y_\alpha = 0, \text{if } U_\alpha \notin C;$

$\therefore Y = \sum_{\alpha=1}^N Y_\alpha = A = \text{No. of members } \in C \text{ in the population.}$

$$P = \frac{A}{N} = \bar{Y}.$$

$n \rightarrow$  The sample size. Sample is drawn by SRSWR.

$y_i = 1, \text{ if } U_i \in C; \quad \text{and} \quad y_i = 0, \text{ if } U_i \notin C;$

$\therefore y = \sum_{i=1}^n y_i = a = \text{No. of members } \in C, \text{ in the sample.}$

$p = \frac{a}{n} = \bar{y} = \text{Sample proportion of members } \in C.$

## Theorem I:

Sample proportion  $p$ , is an unbiased estimator of  $P$ , population proportion.

Proof:

$$p = \frac{\sum_{i=1}^n y_i}{n} = \text{Sample proportion.}$$

$$E(p) = \frac{1}{n} \sum_{i=1}^n E(y_i) = \frac{1}{n} \sum_{i=1}^n \{1.P(U_i \in C) + 0.P(U_i \notin C)\}$$

$$= \frac{1}{n} \sum_{i=1}^n \frac{A}{N} = \bar{Y} = P.$$

Prove that  $V(p) = \frac{P.Q}{n}$ .

**Proof:**

$$\begin{aligned} V(p) &= \frac{1}{n^2} \left[ \sum_{i=1}^n V(y_i) + \sum_{i \neq j} \sum \text{Cov}(y_i, y_j) \right] \\ &= \frac{1}{n^2} \cdot n P Q \\ &= \frac{P Q}{n}. \end{aligned}$$

$$\because V(y_i) = E(y_i^2) - E^2(y_i).$$

$$= P - P^2 = P(1 - P) = PQ.$$

and  $\text{Cov}(y_i, y_j) = 0; \forall i, j; i \neq j$ .

## Theorem II:

An unbiased estimate of the variance of  $p$  is  $v(p) = v(\bar{y}) = \frac{pq}{n-1}$ .

## Proof:

*It is known that  $v(p) = v(\bar{y}) = \frac{s^2}{n}$ .*

$$\begin{aligned} \text{Where, } (n-1)s^2 &= \sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n y_i^2 - n \cdot \bar{y}^2 \\ &= n \cdot p - n \cdot p^2 \\ &= n \cdot p (1 - p) = npq \end{aligned}$$

$$\therefore s^2 = \frac{npq}{n-1}. \quad \therefore v(p) = \frac{npq}{n(n-1)} = \frac{pq}{n-1}.$$

## Theorem II:

p is the unbiased estimate of P, when the sample is drawn by SRSWOR.

Proof:

$$p = \frac{\sum_{i=1}^n y_i}{n} = \text{Sample proportion}$$

$$E(p) = \frac{1}{n} \sum_{i=1}^n E(y_i) = \frac{1}{n} \sum_{i=1}^n \{1.P(U_i \in C) + 0.P(U_i \notin C)\}$$

$$= \frac{1}{n} \sum_{i=1}^n \frac{A}{N} = \bar{Y} = P.$$

Prove that  $V(p) = \frac{N-n}{N-1} \cdot \frac{P.Q}{n}$ .

**Proof:**

$$\begin{aligned} V(p) &= V\{\bar{y}\} = \frac{1}{n^2} V\left\{\sum_{i=1}^n (y_i)\right\} \\ &= \frac{1}{n^2} \left[ \sum_{i=1}^n V(y_i) + \sum_{i \neq j} \sum_{j \neq i} Cov(y_i, y_j) \right] \end{aligned}$$

$$\because V(y_i) = E(y_i^2) - E^2(y_i).$$

$$= P - P^2 = P(1 - P) = PQ.$$

$$Cov(y_i, y_j) = E(y_i y_j) - E(y_i) \cdot E(y_j).$$

$$= \frac{A}{N} \cdot \frac{A-1}{N-1} - P^2 = \frac{P(NP-1) - NP^2 + P^2}{N-1} = - \cdot \frac{PQ}{N-1}$$

$$\therefore V(p) = \frac{1}{n^2} [nPQ + n(n-1)(-\frac{PQ}{N-1})]$$

$$= PQ \left[ \frac{1}{n} - \frac{n-1}{n(N-1)} \right]$$

$$= \frac{PQ}{n} \left[ \frac{N-1-n+1}{(N-1)} \right]$$

$$= \frac{N-n}{N-1} \cdot \frac{PQ}{n}$$

## Theorem III:

$v(p) = \frac{N-n}{N(n-1)} \cdot pq$  is an unbiased estimator of  $V(p)$ .

**Proof:**

Sample variance  $v(p) = v(\bar{y}) = \frac{N-n}{N \cdot n} \cdot s^2$ .

$$(n-1)s^2 = \sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n y_i^2 - n \cdot \bar{y}^2.$$

$$= n \cdot p - n \cdot p^2 = n \cdot p \cdot q. \quad \because \sum_{i=1}^n y_i^2 = \sum_{i=1}^n y_i = a = np.$$

$$v(p) = \frac{N-n}{N(n-1)} \cdot pq$$

$$pq = \frac{\sum_{i=1}^n y_i}{n} \left[ 1 - \frac{\sum_{i=1}^n y_i}{n} \right] = \frac{\sum_{i=1}^n y_i}{n} - \frac{1}{n^2} \left[ \sum_{i=1}^n y_i^2 + \sum_{i \neq j} \sum y_i y_j \right]$$

$$E(pq) = E\left(\frac{\sum_{i=1}^n y_i}{n}\right) - \frac{1}{n^2} [E\{\sum_{i=1}^n y_i^2\} + E\{\sum \sum_{i \neq j} y_i y_j\}]$$

$$= P - \frac{1}{n^2} [nP + n(n-1) \cdot \frac{P(NP-1)}{N-1}]$$

$$= P - \frac{1}{n} [P + (n-1) \cdot \frac{P(NP-1)}{N-1}]$$

$$= P - \frac{1}{n(N-1)} [NP - P + nNP^2 - nP - NP^2 + P]$$

$$= \frac{nNP - nP - NP - nNP^2 + nP + NP^2}{n(N-1)}$$

$$= \frac{nNP(-P+1) - NP(1-P)}{n(N-1)} = \frac{NPQ(n-1)}{n(N-1)}$$

$$\therefore v(p) = \frac{N-n}{N(n-1)} \cdot pq$$

$$\therefore E\{v(p)\} = \frac{N-n}{N(n-1)} \cdot \frac{(n-1)NPQ}{n(N-1)}$$

$$= \frac{PQ}{n} \cdot \frac{N-n}{N-1}$$

$$= V(p)$$

Therefore,  $v(p)$  is an unbiased estimate of  $V(p)$ .

**Think more.....**