GEOMETRIC DISTRIBUTION

Presented
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Definition: A random variable X is said to have a geometric distribution if it assumes only non – negative values and its probability mass function is given by:

$$P(X = x) = pq^{x};\ x = 0,1,2,3,.....;\ 0 < p < 1;\ q = 1 - p$$
$$= 0,\ otherwise$$

Note: Since the various probabilities for x = 0,1,2,3,...... are the terms of geometric progression series, hence the name geometric distribution.

MOMENTS OF GEOMETRIC DISTRIBUTION

$$Mean = \mu'_1 = \sum_{x=0}^{\infty} x.P(X = x) = \sum_{x=0}^{\infty} x.pq^x$$

$$= p[1.q + 2.q^2 + 3.q^3 + \ldots\ldots\ldots\ldots]$$

$$= pq[1 - q]^{-2} = \frac{q}{p}.$$

$$V(X) = E[X - E(X)]^2 = E(X^2) - E^2(X)$$

$$E[X^2] = \sum_{x=0}^{\infty} x^2.P(X = x) = \sum_{x=0}^{\infty} x^2.pq^x$$

$$= p[q + 2^2.q^2 + 3^2.q^3 + 4^2.q^4 + \ldots\ldots\ldots]$$

$$= p.q[1 + 2^2.q + 3^2.q^2 + 4^2.q^3 + ...]$$

$$= p.q(1+q)(1-q)^{-3}$$

$$= \frac{q(1+q)}{p^2}$$

$$V(X) = \frac{q(1+q)}{p^2} - \left(\frac{q}{p}\right)^2$$

$$= \frac{q + q^2 - q^2}{p^2} = \frac{q}{p^2}$$

Problem:

Suppose X is a non-negative integral valued random variable. Show that the distribution of X is geometric if it ‘lacks memory’, i.e., if for each k >= 0 and Y = X-k, one has
P(Y = t / X >= k) = P(X = t).

Proof:

$$Let,\ P(X = r) = p_r\,;\ r = 0,1,2,....$$

$$Define,\ q_k = P(X \geq k) = p_k + p_{k+1} + p_{k+2} + ...$$

$$Given,\ P(Y = t\ /\ X \geq k) = P(X = t) = p_t$$

$$P(Y = t / X \geq k) = \frac{P(Y = t \bigcap X \geq k)}{P(X \geq k)}$$

$$= \frac{P(X = k + t \bigcap X \geq k)}{P(X \geq k)}$$

$$= \frac{P(X = k + t)}{P(X \geq k)} \quad = \frac{p_{k+1}}{q_k}$$

$$\therefore p_t = \frac{p_{k+t}}{q_k}, \ for\ every\ t \geq 0\ and\ all\ k \geq 0.$$

$$When, k = 1\, then$$

$$p_{t+1} = p_t q_1 = (p_1 + p_2 + ...)\, p_t = (1 - p_0)\, p_t$$

$$\Rightarrow p_t = (1-p_0)p_{t-1}$$
$$= (1-p_0)^2 p_{t-2}$$
$$= \ldots\ldots$$
$$= (1-p_0)^t p_0$$

$$Hence,\ p_t = P(X=t) = p_0(1-p_0)^t;\ t = 0,1,2,..$$

$\Rightarrow X$ *has a geometric distribution.*

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