

# **PROBLEMS ON RANDOM VARIABLES**

**Presented  
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**Problem:-1**

For a random variable  $X$ , show that

$[E(X^2)]^{\frac{1}{2}} \geq E(X)$ . Hence, show that mean deviation about mean can not exceed standard deviation.

**Proof:-**

Since,  $\text{Var}(X) \geq 0$ ,

$$\text{So, } E(X^2) - E^2(X) \geq 0$$

$$\text{or, } E(X^2) \geq E^2(X)$$

$$\text{or, } [E(X^2)]^{\frac{1}{2}} \geq E(X)$$

If  $X$  is replaced by  $|X - E(X)|$  in the above proof, then

$$E[|X - E(X)|] \leq \sqrt{E[X - E(X)]^2}.$$

## **Problem:-2**

If  $E(X) = 3$ ,  $E(Y) = 5$ , then find  $E(3X - 5Y + 16)$ .

## **Solution:-**

$$\begin{aligned}E(3X - 5Y + 16) &= 3E(X) - 5E(Y) + 16 \\&= 3 \cdot 3 - 5 \cdot 5 + 16 \\&= 0\end{aligned}$$

## **Problem:-3**

If  $E(X) = 4$ ,  $\text{Var}(X) = 9$ , then find  $E(X^2)$ .

## **Solution:-**

$$\text{Var}(X) = E(X^2) - E^2(X).$$

$$E(X^2) = \text{Var}(X) + E^2(X).$$

$$= 9 + 16$$

## Problem:-4

If  $X$  and  $Y$  are independent, then find  $\text{Cov}(2X, -3Y)$ .

## Solution:-

$$\begin{aligned}\text{Cov}(2X, -3Y) &= E[\{2X - E(2X)\}\{(-3Y) - E(-3Y)\}] \\&= -6.E[\{X - E(X)\}\{(Y) - E(Y)\}] \\&= -6.\text{Cov}(X,Y) \\&= 0\end{aligned}$$

Since,  $X$  and  $Y$  are independent so,  $\text{Cov}(X,Y) = 0$ .

## Problem:-5

If  $E(X) = 5$ ,  $E[X(X-1)] = 44$ , then find  $\text{Var}(1-2X)$ .

## Solution:-

$$\begin{aligned} E[X(X-1)] &= E(X^2) - E(X) \\ &= \text{Var}(X) + E^2(X) - E(X) \\ \therefore \text{Var}(X) &= 44 + E(X) - E^2(X) \\ &= 44 + 5 - 25 \\ &= 24 \end{aligned}$$

$$\text{Var}(1-2X) = 4 \cdot \text{Var}(X) = 4 \cdot 24 = 96$$

## Problem:-6

If a random variable  $X$  assumes only two values -2 and 1 such that  $2.P(X = -2) = P(X = 1)$ , then find  $E(X)$ .

## Solution:-

$$P(X = -2) + P(X = 1) = 1$$

$$\text{Or, } P(X = -2) + 2.P(X = -2) = 1$$

$$\text{Or, } 3.P(X = -2) = 1$$

$$\text{Or, } P(X = -2) = 1/3$$

$$\text{So, } P(X = 1) = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$E(X) = -2.P(X = -2) + 1.P(X = 1)$$

$$= -2 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3} = 0$$

## Problem:-7

If X and Y are independent with  $\text{Var}(X) = 6$  and  $\text{Var}(Y) = 10$ , then find  $\text{Var}(X - Y)$ .

## Solution:-

$$\begin{aligned}\text{Var}(X - Y) &= \text{Var}(X) + \text{Var}(Y) - 2 \text{Cov}(X, Y) \\ &= 6 + 10 = 16\end{aligned}$$

Since, X and Y are independent , so,  $\text{Cov}(X, Y) = 0$ .

## Problem:-8

*If X and Y are both negative random variables , mutually independent,  $E(XY) = 6$  and  $|E(X)| = 2$ , find  $E(Y)$ .*

## Solution:-

$$\begin{aligned}E(XY) &= E(X) \cdot E(Y). \text{ Or, } E(Y) = \frac{E(XY)}{E(X)} \\ \text{Or, } E(Y) &= \frac{6}{-2} = -3. \text{ Since, } E(X) = -2.\end{aligned}$$

## Problem:-9

An unbiased die is thrown twice. Write down the sample space of this experiment. If  $X$  denotes the sum of points obtained in the two throws, obtain the probability distribution of  $X$ .

## Solution:-

Sample space  $S = \{ (1,1), (1,2), (1,3), \dots, (1,6),$   
 $(2,1), (2,2), (2,3), \dots, (2,6),$   
.....  
.....  
 $(6,1), (6,2), (6,3), \dots, (6,6) \}$

$X$  denotes the sum of points obtained in the two throws.

$X:$	2	3	4	5	6	7	8	9
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Prob:	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$
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$X:$	10	11	12	Total
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Prob:	$3/36$	$2/36$	$1/36$	1
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## Problem:-10

A random experiment consists of three independent tosses of a fair coin. Write down the sample space. If  $X$  be the number of heads obtained, obtain the probability distribution of  $X$ . Calculate its expectation and variance.

### Solution:-

$X$  be the number of heads obtained.

Sample space  $S = \{ \text{HHH}, \text{HTH}, \text{THH}, \text{TTH}, \text{HHT}, \text{HTT}, \text{THT}, \text{TTT} \}$

$X$	0	1	2	3	Total
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Prob.	$1/8$	$3/8$	$3/8$	$1/8$	1
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$$E(X) = \sum_{x=0}^3 x.P(X=x) = \frac{1}{8}[0+1.3+2.3+3] = \frac{3}{2}$$

$$E(X^2) = \sum_{x=0}^3 x^2 \cdot P(X = x) = \frac{1}{8}[0 + 1^2 \cdot 3 + 2^2 \cdot 3 + 3^2]$$
$$= \frac{24}{8} = 3$$

$$Var(X) = E(X^2) - E^2(X).$$

$$= 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$$

## Problem:-11

A bag contains 5 white and 3 black balls. 3 balls are drawn randomly without replacement. If  $X$  is a random variable, which takes value 1, if at least 2 white balls are drawn, and value 0, otherwise, find  $E(X)$ .

### Solution:-

X:

0

1

Total

$$\begin{aligned} \text{Prob: } & 1 - \frac{5}{7} \\ & = \frac{2}{7} \end{aligned}$$

$$\frac{5_{C_2} \cdot 3_{C_1}}{8_{C_3}} + \frac{5_{C_3}}{8_{C_3}} = \frac{\frac{5.4}{2} \cdot 3}{\frac{8.7.6}{3.2}} + \frac{\frac{5.4.3}{6}}{\frac{8.7.6}{3.2}} = \frac{40}{56} = \frac{5}{7} \quad 1$$

$$E(X) = \sum_x x \cdot P(X = x) = 0 \cdot \frac{2}{7} + 1 \cdot \frac{5}{7} = \frac{5}{7}$$

## Problem:-12

A perfect coin is tossed 3 times in succession. Given  $X = 1$  if first toss gives head,  $X = 0$  if first toss gives tail and  $Y$  = number of heads obtained in 3 tosses, construct the joint distribution of  $X$  and  $Y$ , and find correlation coefficient between them.

### Solution:-

$Y$	0	1	2	3	Total
$X$					
0	1/8	2/8	1/8	0	1/2
1	0	1/8	2/8	1/8	1/2
Total	1/8	3/8	3/8	1/8	1

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Sample space  $S = \{ HHH, HT\bar{H}, THH, TTH, HHT, HTT, THT, TTT \}$

$$E(X) = \sum_x x \cdot P(X = x) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}.$$

$$E(X^2) = \sum_x x^2 \cdot P(X = x) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}.$$

$$E(Y) = \sum_y y \cdot P(Y = y) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = \frac{12}{8} = \frac{3}{2}.$$

$$E(Y^2) = \sum_y y^2 \cdot P(Y = y) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{1}{8} = \frac{24}{8} = 3.$$

$$Var(X) = E(X^2) - E^2(X) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

$$Var(Y) = E(Y^2) - E^2(Y) = 3 - \frac{9}{4} = \frac{3}{4}.$$

$$E(XY) = \sum_x \sum_y x.y.P(X = x, Y = y)$$

$$= 1.1.\frac{1}{8} + 1.2.\frac{2}{8} + 1.3.\frac{1}{8} = 1.$$

$$Cov(X, Y) = E(XY) - E(X).E(Y) = 1 - \frac{1}{2} \cdot \frac{3}{2} = \frac{1}{4}.$$

$$\rho_{XY} = \frac{Cov(X, Y)}{\sqrt{Var(X).Var(Y)}} = \frac{\frac{1}{4}}{\sqrt{\frac{1}{4} \cdot \frac{3}{4}}} = \frac{1}{\sqrt{3}}.$$

## Problem:-13

If a person gets Rs.  $2X+5$ , where  $X$  denotes the number appearing when a balanced die is rolled once, how much money can he expect in the long run per game?

**Solution:-**

$$E(X) = \sum_{x=1}^6 x \cdot P(X = x)$$

$$= 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$$

$$= \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6] = \frac{7}{2}$$

*He can expect Rs.  $[2E(X) + 5]$  = Rs.  $[2 \cdot \frac{7}{2} + 5]$  = Rs.12.*

## Problem:-14

If  $X$  and  $Y$  are independent random variables with  $E(X) = E(Y) = 0$ , then show that  $V(XY) = V(X).V(Y)$ .

### Solution:-

$$\begin{aligned} V(XY) &= E(X^2 Y^2) - E^2(XY) \\ &= E(X^2)E(Y^2) - E^2(X)E^2(Y) \\ &= E(X^2)E(Y^2) \\ &= E[X - E(X)]^2 E[Y - E(Y)]^2 \\ &= V(X)V(Y) \end{aligned}$$

## Problem:-15

What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability  $p$  of success in each trial?

### Solution:-

Let  $X$  be the number of failures preceding the first success.

$$\begin{aligned} E(X) &= \sum_{x=0}^{\infty} x \cdot p \cdot q^x = p [ 1 \cdot q + 2 \cdot q^2 + 3 \cdot q^3 + \dots ] \\ &= p \cdot q \cdot [ 1 + 2 \cdot q + 3 \cdot q^2 + \dots ] \\ &= p \cdot q \cdot [1 - q]^{-2} = \frac{q}{p}. \end{aligned}$$

## Problem:-16

For two random variables  $X$  and  $Y$ ,

$E(X) = 8, E(Y) = 6, Var(X) = 16, Var(Y) = 36$  and  $\rho_{XY} = 0.5$ .

- Find i)  $E(XY)$ , ii)  $Cov(X, X+Y)$  iii)  $Var(2X - 5Y)$   
iv) Correlation coefficient between  $(2X + 3Y)$  and  $(2X - 3Y)$

## Solution:-

$$\rho_{XY} = \frac{Cov(X, Y)}{\sqrt{Var(X).Var(Y)}}$$

$$Or, Cov(X, Y) = \rho_{XY} \cdot \sqrt{Var(X).Var(Y)}$$

$$Or, E(XY) = 0.5 \cdot \sqrt{16.36} + E(X).E(Y)$$

$$Or, E(XY) = 12 + 48 = 60$$

$$\text{ii) } \text{Cov}(X, X+Y) = \text{Var}(X) + \text{Cov}(X,Y)$$

$$= 16 + 12 = 28$$

$$\text{iii) } \text{Var}(2X - 5Y) = 4.\text{Var}(X) + 25.\text{Var}(Y) - 20.\text{Cov}(X,Y)$$

$$= 4. 16 + 25. 36 - 20. 12 = 724$$

$$\begin{aligned}\text{iv) } \rho_{2X+3Y,2X-3Y} &= \frac{\text{Cov}(2X + 3Y, 2X - 3Y)}{\sqrt{\text{Var}(2X - 3Y).\text{Var}(2X + 3Y)}} \\ &\quad \text{Cov}(2X + 3Y, 2X - 3Y) \\ &= 4.\text{Var}(X) - 6\text{Cov}(X, Y) + 6.\text{Cov}(X, Y) - 9.\text{Var}(Y) \\ &= 4.16 - 9.36 \\ &= 64 - 324 \\ &= -260\end{aligned}$$

$$\begin{aligned}Var(2X - 3Y) &= 4.Var(X) + 9.Var(Y) - 12.Cov(X, Y) \\&= 4 \cdot 16 + 9 \cdot 36 - 12 \cdot 12 = 244\end{aligned}$$

$$\begin{aligned}Var(2X + 3Y) &= 4.Var(X) + 9.Var(Y) + 12.Cov(X, Y) \\&= 4 \cdot 16 + 9 \cdot 36 + 12 \cdot 12 = 532\end{aligned}$$

$$\rho_{2X+3Y, 2X-3Y} = \frac{-260}{\sqrt{244.532}} = \frac{-260}{360.29} = -0.72$$

## Problem:-17

Prove that two uncorrelated random variables are independent, if each of the variables assumes only two distinct values.

### Solution:-

X Y	$x_1$	$x_2$	TOTAL
$y_1$	$P_{11}$	$P_{01} - p_{11}$	$P_{01}$
$y_2$	$P_{10} - p_{11}$	$1 - p_{01} - p_{10} + p_{11}$	$1 - p_{01}$
TOTAL	$P_{10}$	$1 - p_{10}$	1

Since, X and Y are two uncorrelated random variables,  
So,  $\text{Cov}(X,Y) = 0$ .

$$\therefore E(XY) = E(X).E(Y)$$

$$Or, \sum_{i=1}^2 \sum_{j=1}^2 x_i.y_j.P(X = x_i, Y = y_j)$$

$$= \sum_{i=1}^2 x_i.P(X = x_i) \sum_{j=1}^2 y_j P(Y = y_j)$$

$$\begin{aligned} & Or, x_1.y_1.p_{11} + x_1.y_2.(p_{10} - p_{11}) + x_2.y_1(p_{01} - p_{11}) \\ & + x_2.y_2(1 - p_{01} - p_{10} + p_{11}) \\ & = [x_1.p_{10} + x_2.(1 - p_{10})][y_1.p_{01} + y_2.(1 - p_{01})] \end{aligned}$$

*Equating the coefficients of  $x_i \cdot y_j; \forall i, j$ ; from both sides, it is found that*

$$p_{11} = p_{10} \cdot p_{01}$$

$$p_{10} - p_{11} = p_{10} \cdot (1 - p_{01})$$

$$p_{01} - p_{11} = p_{01} \cdot (1 - p_{10})$$

$$1 - p_{01} - p_{10} + p_{11} = (1 - p_{01}) \cdot (1 - p_{10})$$

$$\therefore p_{ij} = P(X = x_i, Y = y_j) = P(X = x_i) \cdot P(Y = y_j)$$

$= p_{i0} \cdot p_{0j}; \forall i, j$ . So,  $X$  and  $Y$  are independent.

## Problem:-18

If  $X$  and  $Y$  are independent random variables such that expectations of  $X$  and  $Y$  are  $\lambda_1$  and  $\lambda_2$  respectively, variances of  $X, Y$  and  $XY$  are  $\sigma_1^2, \sigma_2^2$  and  $\sigma_{12}^2$  respectively, correlation coefficient between  $X$  and  $XY$  is  $\rho_1$  and that between  $Y$  and  $XY$  is  $\rho_2$ , then show that

i)  $\sigma_{12}^2 = \sigma_1^2 \cdot \sigma_2^2 + \lambda_1^2 \cdot \sigma_2^2 + \lambda_2^2 \cdot \sigma_1^2$

ii)  $\rho_1 : \rho_2 = \sigma_1 \lambda_2 : \sigma_2 \lambda_1$

## Proof:

$$\begin{aligned} i) \sigma_{12}^2 &= \text{Var}(XY) = E(X^2Y^2) - E^2(XY) \\ &= E(X^2)E(Y^2) - E^2(X)E^2(Y) \\ &= [\text{Var}(X) + E^2(X)][\text{Var}(Y) + E^2(Y)] - E^2(X)E^2(Y) \\ &= \text{Var}(X)\text{Var}(Y) + \text{Var}(X)E^2(Y) + E^2(X)\text{Var}(Y) \\ &\quad + E^2(X)E^2(Y) - E^2(X)E^2(Y) \\ &= \sigma_1^2 \cdot \sigma_2^2 + \sigma_1^2 \cdot \lambda_2^2 + \sigma_2^2 \cdot \lambda_1^2 \ (\text{Proved}). \end{aligned}$$

$$\begin{aligned}
ii) \frac{\rho_1}{\rho_2} &= \frac{\frac{Cov(X, XY)}{\sqrt{Var(X)Var(XY)}}}{\frac{Cov(Y, XY)}{\sqrt{Var(Y)Var(XY)}}} = \frac{\frac{E(X^2Y) - E(X)E(XY)}{\sigma_1}}{\frac{E(XY^2) - E(Y)E(XY)}{\sigma_2}} \\
&= \frac{E(X^2)E(Y) - E^2(X)E(Y)}{E(X)E(Y^2) - E^2(Y)E(X)} \cdot \frac{\sigma_2}{\sigma_1} \\
&= \frac{E(Y)[E(X^2) - E^2(X)]}{E(X)[E(Y^2) - E^2(Y)]} \cdot \frac{\sigma_2}{\sigma_1} \\
&= \frac{\lambda_2 \cdot \sigma_1^2}{\lambda_1 \cdot \sigma_2^2} \cdot \frac{\sigma_2}{\sigma_1} = \frac{\lambda_2 \cdot \sigma_1}{\lambda_1 \cdot \sigma_2} (\text{Proved}).
\end{aligned}$$

## Problem:-19

If two random variables  $X$  and  $Y$  such that  $E(X) + E(Y) = 0$ ,  $\text{Var}(X) - \text{Var}(Y) = 0$  and  $1 + \rho_{XY} = 0$ , what is the relationship between  $X$  and  $Y$ .

### Solution:-

Let,  $Y = a + b.X$ . So,  $E(Y) = a + b.E(X)$

Again,  $E(X) + E(Y) = 0$ . So,  $a = 0$  and  $b = -1$ .

Therefore,  $Y = -X$ . Hence,  $\text{Var}(Y) = \text{Var}(X)$ .

## Problem:-20

If X,Y and Z are three random variables so that X and Y are independent and  $Z=XY$ . X can take two values 10 and 20, and the probability that X is 10 is  $1/3$ . Y can take three values 5,6 and 7. The probability that Y is 5 is  $\frac{1}{4}$  and that it is 6 is  $\frac{1}{2}$ . Find the expectation of Z.

## Solution:-

X	10	20	Total
Probability	$1/3$	$2/3$	1

Y	5	6	7	Total
Probability	1/4	1/2	1/4	1

$$E(X) = \sum_x x.P(X = x) = 10 \cdot \frac{1}{3} + 20 \cdot \frac{2}{3} = \frac{50}{3}$$

$$E(Y) = \sum_y y.P(Y = y) = 5 \cdot \frac{1}{4} + 6 \cdot \frac{1}{2} + 7 \cdot \frac{1}{4} = 6$$

$$E(Z) = E(XY) = E(X).E(Y) = \frac{50}{3} \cdot 6 = 100$$

## Problem:-21

Two random variables  $X$  and  $Y$  are jointly distributed such that  $P(Y = i) = \frac{1}{4}(2i - 1)$ , for  $i = 1, 2$ ;  $P(X = i) = \frac{1}{2}$ , for  $i = 0, 1$ ; and  $P(X = 0, Y = 2) = p$ . Find out the correlation coefficient between  $X$  and  $Y$ , and find the interval in which possible values of  $p$  lie. For what value of  $p$ , correlation coefficient is zero? Are then  $X$  and  $Y$  independent?

## Solution:-

X \ Y	1	2	TOTAL
X			
0	$\frac{1}{2} - p$	$p$	$1/2$
1	$\frac{1}{2} - \frac{3}{4} + p$	$\frac{3}{4} - p$	$1/2$
TOTAL	$1/4$	$3/4$	1

$$E(X) = \sum_x x \cdot P(X = x) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}.$$

$$E(X^2) = \sum_x x^2 \cdot P(X = x) = 0 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{2} = \frac{1}{2}.$$

$$E(Y) = \sum_y y \cdot P(Y = y) = 1 \cdot \frac{1}{4} + 2 \cdot \frac{3}{4} = \frac{7}{4}.$$

$$E(Y^2) = \sum_y y^2 \cdot P(Y = y) = 1 \cdot \frac{1}{4} + 2^2 \cdot \frac{3}{4} = \frac{13}{4}.$$

$$\begin{aligned} E(XY) &= \sum_x \sum_y x \cdot y \cdot P(X = x, Y = y) \\ &= 1 \cdot 1 \cdot \left( \frac{1}{2} - \frac{3}{4} + p \right) + 1 \cdot 2 \cdot \left( \frac{3}{4} - p \right) \end{aligned}$$

$$= p - \frac{1}{4} + \frac{6}{4} - 2p = \frac{5}{4} - p$$

$$Cov(X, Y) = E(XY) - E(X).E(Y) = \frac{5}{4} - p - \frac{1}{2} \cdot \frac{7}{4} = \frac{3}{8} - p.$$

$$Var(X) = E(X^2) - E^2(X) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

$$Var(Y) = E(Y^2) - E^2(Y) = \frac{13}{4} - \frac{49}{16} = \frac{52 - 49}{16} = \frac{3}{16}.$$

$$\rho_{XY} = \frac{Cov(X, Y)}{\sqrt{Var(X).Var(Y)}} = \frac{\frac{3}{8} - p}{\sqrt{\frac{1}{4} \cdot \frac{3}{16}}} = \frac{3 - 8p}{\sqrt{3}}.$$

$$\rho_{XY} = 0. Or, 3 - 8p = 0 Or, p = \frac{3}{8}.$$

X Y	1	2	TOTAL
0	$1/2 - 3/8 = 1/8$	$3/8$	$1/2$
1	$1/2 - 3/4 + 3/8 = 1/8$	$3/4 - 3/8 = 3/8$	$1/2$
TOTAL	$1/4$	$3/4$	1

$$\therefore P(X = x_i, Y = y_j) = P(X = x_i).P(Y = y_j);$$

$\forall i, j.$  So,  $X$  and  $Y$  are independent.

$$\frac{1}{2} - p \geq 0 \Rightarrow p \leq \frac{1}{2}.$$

$$p - \frac{1}{4} \geq 0 \Rightarrow p \geq \frac{1}{4}. \text{ Again, } \frac{3}{4} - p \geq 0 \Rightarrow p \leq \frac{3}{4}.$$

$$\text{So, } \frac{1}{4} \leq p \leq \frac{1}{2}.$$

**Problem:-22**

*Obtain the median of the distribution given by the p.d.f.*

$$f(x) = 3x^2, 0 \leq x \leq 1$$

*Find 'a' such that  $P(X > a) = 0.05$ .*

**Solution:-**

*Let median be 'm'.*

$$\therefore \int_0^m f(x)dx = \frac{1}{2} \quad or, \int_0^m 3x^2 dx = \frac{1}{2} \quad or, [x^3]_0^m = \frac{1}{2}$$

$$or, m^3 = \frac{1}{2} \quad or, m = \sqrt[3]{\frac{1}{2}}$$

$$P(X > a) = 0.05$$

$$\text{or, } \int_a^1 3x^2 dx = 0.05$$

$$\text{or, } [x^3]_a^1 = 0.05$$

$$\text{or, } a^3 = 1 - 0.05 = 0.95$$

$$\text{or, } a = 0.983$$

## Problem:-23

*A number is chosen at random from the set {1,2,...,100}, and another is chosen independently and at random from the set {1,2,...,50}. What is the expected value of the product of the two numbers.*

## Solution:-

*Let, X be a number is chosen from the first set and Y be a number is chosen from the second.*

$$P(X = x) = \frac{1}{100}, x = 1, 2, \dots, 100$$

$$P(Y = y) = \frac{1}{50}, y = 1, 2, \dots, 50$$

$$E(X) = \sum_{x=1}^{100} x \cdot P(X = x) = \frac{1}{100} [1 + 2 + \dots + 100]$$

$$= \frac{100(100+1)}{100 * 2} = \frac{101}{2} = 50.5$$

$$E(Y) = \sum_{y=1}^{50} y \cdot P(Y = y) = \frac{1}{50} [1 + 2 + \dots + 50]$$

$$= \frac{50(50+1)}{50 * 2} = \frac{51}{2} = 25.5$$

$$E(Z = X * Y) = E(X) * E(Y) = 50.5 * 25.5 = 1287.75$$

# THANK YOU

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