

## **EXPONENTIAL DISTRIBUTION**

**PRESENTED  
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**Definition:**

*A random variable  $X$  is said to have an exponential distribution with parameter  $\theta > 0$ , if its p.d.f. is given by :*

$$\begin{aligned}f(x) &= \theta \cdot e^{-\theta x}, x \geq 0 \\&= 0, \text{ otherwise}\end{aligned}$$

## MOMENT GENERATING FUNCTION

$$M_x(t) = E(e^{tX}) = \int_0^{\infty} e^{tx} \cdot \theta \cdot e^{-\theta x} dx$$

$$= \theta \int_0^{\infty} e^{-(\theta-t)x} dx = \frac{\theta}{(\theta-t)} \int_0^{\infty} e^{-u} \cdot u^{1-1} du$$

Let,  $(\theta - t)x = u$

$$\therefore dx = \frac{du}{(\theta - t)}$$

$$= \left(1 - \frac{t}{\theta}\right)^{-1} = \sum_{r=0}^{\infty} \left(\frac{t}{\theta}\right)^r, \theta > t$$

$$\therefore \mu'_r = E(X^r) = \text{Coefficient of } \frac{t^r}{r!} \text{ in } M_x(t) = \frac{r!}{\theta^r}; r = 1, 2, \dots$$

$$\text{Mean} = \mu'_1 = \frac{1}{\theta} \text{ and variance } \mu_2 = \mu'_2 - \mu'^2_1 = \frac{2}{\theta^2} - \frac{1}{\theta^2} = \frac{1}{\theta^2}$$

## MOMENTS

*rth order raw moment*  $\mu'_r = E(X^r) = \int_0^\infty x^r \cdot f(x) dx$

$$= \int_0^\infty x^r \cdot \theta \cdot e^{-\theta x} dx = \int_0^\infty \left(\frac{u}{\theta}\right)^r \cdot \theta \cdot e^{-u} \frac{du}{\theta}$$

Let,  $\theta \cdot x = u$   
 $\therefore dx = \frac{du}{\theta}$

$$= \frac{1}{\theta^r} \int_0^\infty u^{(r+1)-1} \cdot e^{-u} du = \frac{\Gamma(r+1)}{\theta^r}$$

*Mean*  $\mu'_1 = E(X) = \frac{\Gamma 2}{\theta} = \frac{1}{\theta}$  and  $\mu'_2 = \frac{\Gamma 3}{\theta^2} = \frac{2}{\theta^2}$ .

$$V(X) = \mu'_2 - \mu'^2_1 = \frac{2}{\theta^2} - \frac{1}{\theta^2} = \frac{1}{\theta^2}.$$

## CUMULATIVE DISTRIBUTION FUNCTION

$$F(x) = P(X \leq x) = \int_0^x f(t)dt = \int_0^x \theta \cdot e^{-\theta t} dt$$

$$= \left[ \frac{\theta \cdot e^{-\theta t}}{-\theta} \right]_0^x = 1 - e^{-\theta x}$$

$$F(Q_3) = P(X \leq Q_3) = \frac{3}{4} \quad \text{or, } 1 - e^{-\theta \cdot Q_3} = \frac{3}{4}$$

$$\text{or, } e^{-\theta \cdot Q_3} = \frac{1}{4} \quad \text{or, } \theta \cdot Q_3 = \ln 4 \quad \text{or, } Q_3 = \frac{1}{\theta} \ln 4$$

$$F(Q_1) = P(X \leq Q_1) = \frac{1}{4} \quad \text{or, } 1 - e^{-\theta \cdot Q_1} = \frac{1}{4}$$

$$\text{or, } e^{-\theta \cdot Q_1} = \frac{3}{4} \quad \text{or, } \theta \cdot Q_1 = \ln 4 - \ln 3$$

$$\text{or, } Q_1 = \frac{1}{\theta} [\ln 4 - \ln 3]$$

$$\therefore \text{Quartile Deviation}(Q.D.) = \frac{Q_3 - Q_1}{2}$$

$$= \frac{\ln 4 - \ln 4 + \ln 3}{2\theta} \quad = \frac{1}{2\theta} \cdot \ln 3$$

## LACK OF MEMORY PROPERTY

Show that the exponential distribution 'lacks memory', i.e. if  $X$  has an exponential distribution, then for every constant  $a \geq 0$ , one has

$$P(Y \leq x / X \geq a) = P(X \leq x), \forall x, \text{ where } Y = X - a$$

*Solution :* The p.d.f. of the exponential distribution with parameter  $\theta$  is :

$$\begin{aligned} f(x) &= \theta \cdot e^{-\theta x}, x \geq 0, \theta > 0 \\ &= 0, \text{ otherwise} \end{aligned}$$

$$P(Y \leq x \cap X \geq a) = P(X - a \leq x \cap X \geq a)$$

$$P(a \leq X \leq a+x) = F(a+x) - F(a)$$

$$\begin{aligned} &= 1 - e^{-\theta(a+x)} - 1 + e^{-\theta a} \\ &= e^{-\theta \cdot a} [1 - e^{-\theta \cdot x}] \end{aligned}$$

$$P(Y \leq x / X \geq a) = \frac{P(Y \leq x \cap X \geq a)}{P(X \geq a)}$$

$$= \frac{e^{-\theta \cdot a} [1 - e^{-\theta \cdot x}]}{1 - [1 - e^{-a \cdot \theta}]} = 1 - e^{-\theta \cdot x} = P(X \leq x)$$

*So, exponential distribution lacks memory.*

**THANKS FOR YOUR PATIENCE**